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AUTHOR(S):

Shimizu, Yuji

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D-modules on the moduli spaces of curves associated with abelian CFT

Yuji SHIMIZU
Mathematical Institute,
Faculty of Science,
Tohoku University

§ Introduction

This report is a short review of a work in progress, jointly with Prof. Kenji Ueno, on abelian conformal field theory (abbreviated as CFT). The adjective "abelian" means that we start from the symmetry of the gauge group $\mathbf{G}_m = \mathbf{C}^*$ (the multiplicative group) in contrast to the "non-abelian" CFT's.

Our motivation is to give a framework, natural from the geometric viewpoint, to this abelian CFT studied by [IMO], [KNTY], [ACKP], etc, in a fashion comparable to [BS], [TUY].

The technique of localization is developed by [BS], [BFM], based on the idea that the dressed moduli spaces are a kind of flag varieties for Virasoro and affine Lie algebras, cf.[K].

CFT tells how to construct (projective) monodromy representations of the Teichmüller modular group (i.e. mapping class group). We want to study these monodromy representations and the corresponding systems of differential equations are called as (D-)modules of conformal blocks. Localization procedure produces these modules from the given representations of the (infinite-dimensional) Lie algebra.

We will treat the case of the affinization of the one-dimensional Lie algebra (the $U(1)$-current algebra). We describe the construction of the D-modules of conformal blocks and comment on the factorization property.

Our consideration can be extended to the non-abelian case to a certain extent. It will be treated in our work in preparation.

§1 Setting

1.1 "Moduli"

We fix non-negative integers g, N with $3g-3+N \geq 0$. A scheme means a $\mathbf{C}$-scheme in what follows.

Let $\mathcal{M}_{g,N}$ be the moduli space of N-pointed smooth projetive algebraic curves over $\mathbf{C}$ of genus g, and $\overline{\mathcal{M}}_{g,N}$ its natural compactification, i.e., the moduli space of N-pointed stable curves of genus g. These are smooth stacks of dimension $3g-3+N$ and $\overline{\mathcal{M}}_{g,N}$ is also proper over $\mathbf{C}$, cf.[DM],[Kn].

Consider the morphism

$$\pi : \overline{\mathcal{C}} = \overline{\mathcal{M}}_{g,N+1} \to \overline{\mathcal{M}} = \overline{\mathcal{M}}_{g,N}$$

which forgets the $(N+1)$-th point and is the "univesal" curve. By restriction this gives rise to the universal curve

$$\pi : \mathcal{C} = \mathcal{C}_{g,N} \to \mathcal{M} = \mathcal{M}_{g,N}.$$

We can consider the relative Picard group for π of degree $g-1$ $\overline{P} = Pic^{g-1}_{\overline{\mathcal{C}}/\overline{\mathcal{M}}}$, which is a semi-abelian scheme over $\overline{\mathcal{M}}$. Its restriction to $\mathcal{M}$ is a (relative) abelian scheme of dimension g over $\mathcal{M}$. Put $P = Pic^{g-1}_{\mathcal{C}/\mathcal{M}} \xrightarrow{p} \mathcal{M}$. We have the Poincaré bundle $\mathcal{P}$ on $\overline{\mathcal{C}} \times_{\overline{\mathcal{M}}} \overline{P}$ and dispose the determinant line bundle $\mathcal{L} = d(\mathcal{P}) = \det \mathcal{R}\pi_*(\mathcal{P})$ on $\overline{P}$, where π is the projection $\overline{\mathcal{C}} \times_{\overline{\mathcal{M}}} \overline{P} \to \overline{P}$. cf.[KM].

1.2 " Dressing"

Let $\mathcal{M}^{(\infty)}_{g,N}$ be the moduli space of dressed N-pointed curves, which is introduced by Beilinson and Kontsevich and which we call the dressed moduli space, cf.[KNTY,KSU]. A dressed N-pointed curve $(C; x_1, \cdots, x_N; t_1, \cdots, t_N)$ consists of a N-pointed curve $(C; x_1, \cdots, x_N)$ and isomorphisms of $\mathbf{C}$-algebras $t_i : \widehat{\mathcal{O}}_{C,x_i} \simeq \mathbf{C}[[z]] (1 \leq i \leq N)$ (formal local parametrizations).

Put $\mathcal{M}^{(\infty)} = \mathcal{M}^{(\infty)}_{g,N}$ and $P^{(\infty)} = P \times_{\mathcal{M}} \mathcal{M}^{(\infty)}$.

Consider also the moduli space of dressed invertible sheaves over dressed N-pointed curves $P^{(\#)}$. A dressed invertible sheaf on a dressed N-pointed curve $(C; x_1, \cdots, x_N; t_1, \cdots, t_N)$ is an invertible sheaf L (or a line bundle) equipped with t_i-linear isomorphisms $v_i : \widehat{L}_{x_i} \simeq \mathbf{C}[[z]] (1 \leq i \leq N)$. Thus $P^{(\#)}$ is a $\mathbf{G}_m(\mathbf{C}[[z]])^N$-torsor over $P^{(\infty)}$.

1.3 Let us summarize the situation in the following "basic diagram" :

$$\begin{array}{cccc} & r^*\mathcal{L} \to & P^{(\#)} & \\ & r \swarrow & & \\ \mathcal{L} \to P^{(\infty)} = Pic^{g-1}_{C/\mathcal{M}^{(\infty)}} & & \downarrow q & \\ & p \searrow & & \\ & & \mathcal{M}^{(\infty)} = & \mathcal{M}^{(\infty)}_{g,N} \to \mathcal{M} = \mathcal{M}_{g,N} \end{array}$$

We have a similar picture over $\overline{\mathcal{M}}_{g,N}$.

1.4 "Some representation theory"

We recall some notations for representations of an infinite-dimensional Heisenberg algebra.

Let $u(1)$ be the one-dimensional Lie algebra over $\mathbf{C}$ and

$$\hat{u}(1) = \mathbf{C}((z)) \oplus \mathbf{C} \cdot K, \quad \mathbf{C}((z)) = \mathbf{C}[[z]][z^{-1}]$$

be its (completed) affinization, where K is a central element. Its Lie bracket is defined to be

$$[f(z), g(z)] = \operatorname{Res}_{z=0}(f'(z)g(z)dz) \cdot K$$

for $f(z), g(z) \in \mathbf{C}((z))$. Thus, for example, we have

$$[z^m, z^n] = m\delta_{m+n,0} \cdot K.$$

Therefore $\hat{u}(1)$ contains an infinite-dimensional Heisenberg algebra generated by the elements $h_n = z^n, h_{-n} = z^{-n}/n\,(n > 0), h_0 = K$.

We also consider the "N-point" variant of $\hat{u}(1)$:

$$\hat{u}_N(1) = \bigoplus_{i=1}^{N} \mathbf{C}((z_i)) \oplus \mathbf{C} \cdot K.$$

Here K is again a central element and the Lie bracket is given by

$$[(f_i(z_i)), (g_i(z_i))] = \sum_{i=1}^{N} \operatorname{Res}_{z_i=0}(f_i'(z_i)g_i(z_i)dz_i) \cdot K.$$

Put also

$$u_N(1) := \bigoplus_{i=1}^{N} \mathbf{C}((z_i)).$$

Then, $\hat{u}_N(1)$ is a central extension of $u_N(1)$ as a Lie algebra.

Let us introduce the Fock representations which have two complex parameters λ, w :

$$F(w,\lambda) := U(\hat{u}(1))/I(w,\lambda),$$

where $U(\hat{u}(1))$ is the universal enveloping algebra of $\hat{u}(1)$ and $I(w,\lambda)$ is the left ideal of $U(\hat{u}(1))$ generated by $\mathbf{C}[[z]]$, $z^0 - w$ and $K - \lambda$.

Put $F_0 := F(0,0)$ for later use.

Given $\lambda, w_1, \cdots, w_N \in \mathbf{C}$. Then we have natuturally an action of $\hat{u}_N(1)$ on the tensor product $F(\lambda, \vec{w}) := \otimes_{i=1}^N F(\lambda, w_i)$ ($\vec{w} = (w_1, \ldots, w_N)$).

§2 Localization

Localization is a natural procedure for producing the space of coinvariants as a fiber of a D-module. It is effectively used in the study of representations of finite-dimensional reductive Lie groups by Beilinson-Bernstein and Brylinski-Kashiwara.

2.1 PROPOSITION

There exists a Lie algebra homomorphism with dense image :

$$\theta : \mathcal{O}_{\mathcal{P}(*)} \otimes_{\mathbf{C}} \hat{u}_N(1) \to D^{\leq 1}_{r^*\mathcal{L}/\mathcal{M}(\infty)}$$

which lifts a natural Lie algebra homomorphism (with dense image) :

$$\theta : \mathcal{O}_{\mathcal{P}(*)} \otimes_{\mathbf{C}} u_N(1) \to T_{P(*)/\mathcal{M}(\infty)}.$$

Both θ have the same kernel :

$$\mathrm{Ker}\theta = \pi_* \mathcal{O}_C(* \sum s_i).$$

There is a similar Lie algebra homomorphism with $\mathcal{L}$ replaced by $\mathcal{L}^{-1}$.

We denote by $D_{r^*\mathcal{L}/\mathcal{M}(\infty)}$ the (shear of) ring of differential operators acting on the sections of the invertible sheaf $r^*\mathcal{L}$ which commute with $\mathcal{O}_{\mathcal{M}(\infty)}$, and $D^{\leq 1}_{r^*\mathcal{L}/\mathcal{M}(\infty)}$ denotes the part of operators of order ≤ 1. For more on these notions, see 3.2.

For the construction of $\theta : \mathcal{O}_{\mathcal{P}(*)} \otimes_{\mathbf{C}} u_N(1) \to T_{P(*)/\mathcal{M}(\infty)}$, we make use of the following short exact sequence :

$$0 \to \mathcal{E}nd(L)(-(m+1)\sum x_i) \to \mathcal{E}nd(L)(n \sum x_i) \to \oplus_{i=1}^N \oplus_{k=-n}^m \mathbf{C} z_i^k \to 0.$$

Here $\mathcal{X} = (C; x_1, \cdots, x_N; t_1, \cdots, t_N; L; v_1, \cdots, v_N)$ denotes a point of $P^{(\#)}$ and the sheaf homomorphism $\mathcal{E}nd(L)(n \sum x_i) \to \oplus_{i=1}^{N} \oplus_{k=-n}^{m} \mathbf{C} z_i^k$ is induced from v_i's. The associated coboundary map (from H^0 to H^1) gives rise to the fiber at $\mathcal{X}$ of θ.

For the construction of its lifting θ, we use a theorem of Beilinson-Schechtman [BS, Thm.2.3]. It is essentially a kind of Čech calculation around the sections.

Let us denote by the same symbol θ the composition of the above θ and the natural inclusion $D^{\leq 1}_{r^\bullet \mathcal{L}/\mathcal{M}} \hookrightarrow D^{\leq 1}_{r^\bullet \mathcal{L}}$:

$$\theta : \mathcal{O}_{P(\#)} \otimes_{\mathbf{C}} \hat{u}_N(1) \to D^{\leq 1}_{r^\bullet \mathcal{L}}$$

This gives rise to the following algebra homomorphism :

$$\theta : \mathcal{O}_{P(\#)} \otimes_{\mathbf{C}} U(\hat{u}_N(1)) \to D_{r^\bullet \mathcal{L}}.$$

2.2 DEFINITION Given a representation E of $\hat{u}_N(1)$, we define its localization $\Delta(E)$ by

$$\Delta(E) := D_{r^\bullet \mathcal{L}} \otimes_{U(\hat{u}_N(1))} E.$$

Let F_0 denote the Fock representation $F(0,0)$ and put

$$\Delta(F_0^{\otimes N}) = M.$$

(Note that $F_0^{\otimes N} = F(0, \vec{0})$ in the notation of 1.4.)

This is just the D-module of gauge condition according to the following :

2.3 LEMMA

We have

$$\Delta(F(\lambda, \vec{w})) \otimes_{\mathcal{O}_{P(\#)}} \mathcal{O}_{P(\#)} / m_{\mathcal{X}} \cdot \mathcal{O}_{P(\#)} \simeq F(\lambda, \vec{w}) / H^0(C, \mathcal{E}nd(L)(* \textstyle\sum x_i)) F(\lambda, \vec{w}).$$

Note that $H^0(C, \mathcal{E}nd(L)$, considered as a subspace of $\bigoplus_{i=1}^{N} \mathbf{C}((z_i)) \subset \hat{u}_N(1)$ by means of local parametrizations t_i, is a Lie subalgebra of $\hat{u}_N(1)$, since we have

$$[f,g]=\sum_{i=1}^{N}\mathrm{Res}_{z_i=0}(v_i(f)'v_i(g)dz_i)=\sum_{i=1}^{N}\mathrm{Res}_{x_i}(gdf)=0.$$

It is obvious that $\Delta(E)$ is a coherent $D_{\tau^*\mathcal{L}}$-module if E is finitely generated as a $U(\hat{u}_N(1))$-module.

In parallel with [TUY], we call the space

$$\mathcal{V}_{\lambda,\vec{w}}(\mathcal{X}):=F(\lambda,\vec{w})/H^0(C,\mathcal{E}nd(L)(*\sum x_i))F(\lambda,\vec{w})$$

the space of covacua attached to $\mathcal{X}$.

The vector space $\mathcal{V}^{\dagger}_{\lambda,\vec{w}}(\mathcal{X})$ dual to $\mathcal{V}_{\lambda,\vec{w}}(\mathcal{X})$ is called the space of vacua. The following spaces are of particular interest :

$$\mathcal{V}(\mathcal{X}):=\mathcal{V}_{0,\vec{0}}(\mathcal{X}),\quad \mathcal{V}^{\dagger}(\mathcal{X}):=\mathcal{V}^{\dagger}_{0,\vec{0}}(\mathcal{X}).$$

2.4 Let us define the "dual" of localization, which looks natural from the viewpoint of representation theory.

Let M^* be defined as the $\mathcal{O}_{P(\sharp)}$- submodule of $\mathcal{O}_{P(\sharp)}\otimes_{\mathbf{C}}(F_0^{\dagger})^{\widehat{\otimes}N}$ annihilated by $\pi_*\mathcal{O}_C(*\sum s_i)$:

$$M^*:=\{\langle u|\in\mathcal{O}_{P(\sharp)}\otimes_{\mathbf{C}}(F_0^{\dagger})^{\widehat{\otimes}N}\mid F\cdot\langle u|=0\quad\text{for}\quad F\in\pi_*\mathcal{O}_C(*\sum s_i)\}$$

From Proposition 2.1, we infer easily the following lemma.

2.5 Lemma

M^* is a $D_{\tau^*\mathcal{L}^{-1}}$-module.

With respect to the natural pairing between F_0 and $F_0^{\dagger}$, the fiber of M^* is equal to the space of vacua :

$$M^*\otimes_{\mathcal{O}_{P(\sharp)}}\mathcal{O}_{P(\sharp)}/m_{\mathcal{X}}\cdot\mathcal{O}_{P(\sharp)}\simeq\mathcal{V}^{\dagger}(\mathcal{X}).$$

§3 Modules of conformal blocks

3.1 Consider the $\mathcal{O}_{P(\infty)}$-module N defined as follows :

$$N=(r_*M)^{\prod_{i=1}^{N}\mathrm{G}_m(\widehat{\mathcal{O}}_{s_i})}.$$

Then N has a structure of $D_{\mathcal{L}}$-module inherited from that of $D_{\tau^*\mathcal{L}}$-module on M.

We define the module of conformal blocks to be the direct image p_*N, which has a priori a structure of $p_*D_{\mathcal{L}}$-module. This module can be given a structure of twisted D-module in a simple manner as found by Beilinson and Kazhdan [BK], which we recall in 3.3.

We can formulate the dual version N^* of N which has a structure of $D_{\mathcal{L}^{-1}}$-module.

3.2 We recall the definition of a ring of twisted differential operators (abbreviated as a tdo).

A tdo is a filtered ring (i.e. sheaf of rings) (D, F), which satisfies the following conditions :

i) $\cup_i F_i D = D, \quad F_{-1} D = 0.$

ii) $F_i D / F_{i-1} D \simeq S^i(T_X)$

compatibly with the multiplications on the both sides.

If $\mathcal{L}$ is a line bundle on X, then the sheaf $D_{\mathcal{L}}$ of differential operators acting on $\mathcal{L}$ is a basic example of tdo. The sheaf $D_{\tau^*\mathcal{L}/\mathcal{M}^{(\infty)}}$ in 2.1 is a family of tdo's parametrized by $\mathcal{M}^{(\infty)}$.

Let us recall a theorem of Beilinson and Kazhdan [BK] mentioned in 3.1 :

3.3 LEMMA

Let $f : X \to S$ be a proper smooth morphism between smooth schemes whose fiber has a structure of abelian variety. Let also $\mathcal{L}$ be an relative invertible sheaf on X whose restriction to fibers is not of the same class as half the canonical class in the Picard group of the fiber.

Then $f_* D_{\mathcal{L}}$ is also a tdo.

Thanks to this lemma, $p_* D_{\mathcal{L}}$ is a tdo. As to $p_* D_{\mathcal{L}^{-1}}$, it is a tdo and is isomorphic to $D_{\frac{1}{2}\lambda_H}$, where λ_H is the Hodge line bundle, i.e., the determinant line bundle det $R\pi_*\mathcal{O}$. This follows from the fact that $p_*\mathcal{L}^{-1}$ is isomorphic to $\frac{1}{2}\lambda_H$ on $\mathcal{M}_g$.

In conclusion, $p_* N^*$ has a structure of $D_{\frac{1}{2}\lambda_H}$-module. This means that we have a natural (twisted) integrable connection on $p_* N^*$. This connection can be interpreted as a kind of heat equation cf.§4.

3.4 "Plücker embedding"

The determinant line bundle $\mathcal{L}$ is known to equal $\mathcal{O}(-\Theta)$ for the theta divisor Θ on $Pic^{g-1}_{C/\mathcal{M}}$, cf.[Sz]. We relate $\mathcal{L}$ or its inverse to the structure of the modules of conformal blocks.

Let us calculate a fiber of the determinant line bundle $\mathcal{L}^{-\infty}$. Let $\mathcal{X} = (C; x_1, \ldots, x_N; t_1, \ldots, t_N; L; v_1, \ldots, v_N)$ be a point of $P^{(\#)}$. Then the fiber at $\mathcal{X}$ of $r^*\mathcal{L}^{-1}$ is isomorphic to $d(\omega_C \otimes L^{-1}) := \det R\Gamma(C, \omega_C \otimes L^{-1})$ where ω_C denotes the dualizing sheaf of the curve C. From the exact sequence

$$0 \to \omega_C \otimes L^{-1} \to \omega_C \otimes L^{-1}(m \sum_i x_i) \to \oplus_{i=1}^N \oplus_{k=-1}^{-m} \mathbf{C} z_i^k dz_i \to 0$$

we have

$$d(\omega_C \otimes L^{-1}) = d(\omega_C \otimes L^{-1}(m \sum_i x_i)) \cdot \wedge^{\max}(\oplus_{i=1}^N \oplus_{k=-1}^{-m} \mathbf{C} z_i^k dz_i)^{-1}.$$

Note that the data t_i's and v_i's are necessary here.

Recall that $F_0^\dagger$ is realized as a semi-infinite form module, which is obtained from the vector space $\mathbf{C}((z))$ by semi-infinite wedge product, cf.[KNTY,§1].

In our situation, the isomorphisms v_i 's induce an embedding :

$$H^0(C, L^{-1} \otimes \omega_C(* \sum x_i)) \hookrightarrow \bigoplus_{i=1}^N \mathbf{C}((z_i)).$$

Then, it defines a line in $(F_0^\dagger)^{\widehat{\otimes} N}$ by the semi-infinite wedge product. This leads to the following :

3.5 Lemma

There exists a natural embedding

$$r^*\mathcal{L}^{-1} \hookrightarrow M^*$$

of $D_{r^*\mathcal{L}^{-1}}$-modules.

This gives rise to the natural embedding

$$\mathcal{L}^{-1} \hookrightarrow N^*.$$

Hence we have

$$p_*\mathcal{L}^{-1} \hookrightarrow p_*N^*.$$

We have similar construction for their "duals".

The basic problem is to understand the structure of p_*N or p_*N^*. This can be done by the above embedding of $p_*\mathcal{L}$ or $p_*\mathcal{L}^{-1}$ and the consideration of theta structures, cf.[SU].

§4 Comments on further results and problems

4.1 "Line bundles on moduli"

To formulate the factorization property for our D-modules of conformal blocks, we need to know the boundary behaviour of the basic line bundle $\mathcal{L}$.

Consider the diagram :

$$\begin{array}{ccccc} \overline{P}_g & \supset P^b & & \sigma^* P^b & \overset{\varpi}{\to} P_{g-1} \\ p\downarrow & \downarrow & & \downarrow & \swarrow \\ \overline{\mathcal{M}}_{g,N} & \supset D_0 & \simeq \overline{\mathcal{M}}_{g-1,N+2}/(\mathbf{Z}/2) \overset{\sigma}{\leftarrow} & \overline{\mathcal{M}}_{g-1,N+2} & \end{array}$$

Here D_0 is the open dense subset corresponding to smooth curves of the irreducible divisor of $\overline{\mathcal{M}}_{g,N}$ whose general point represents an irreducible curve with only one node. The left square is cartesian and ϖ has a structure of $\mathbf{C}^*$-bundle.

Then we have

PROPOSITION

$$\sigma^* \mathcal{L}_g|_{D_0} \simeq \varpi^* \mathcal{L}_{g-1}.$$

This result is analogous to the theorem of Beilinson and Manin which states that the restriction of the Hodge line bundle to D_0 is again the Hodge line bundle of the genus less by one.

Using the above result, we can formulate the factorization property of the conformal blocks using the nearby cycle functor along the boundary D_0. We develop necessary techniques for this purpose such as the nearby cycle functor for twisted D-modules, correspondence with monodromic D-modules on the total space of a line bundle, etc.

We have to care about the compactification of our Picard schemes and D-modules on (singular) algebraic stacks.

4.2 We end this exposition by commenting on two points.

The first is the problem of descent of the D-modules of conformal blocks. It is closely connected with the so-called Sugawara construction for a $U(1)$-current algebra. We can verify the compatibility of localizations and the Sugawara construction. The latter essentially reduces to the heat equation.

The second concerns the structure of $p_* N^{(*)}$ as mentioned in 3.5. We can analyze its structure in terms of Heisenberg group by introducing the moduli space of pointed curves with theta characteristics.

For these, we refer to [SU].

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